The Cosmological Constant in Causal Set Theory

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Overview

- The cosmological constant
- The causal set approach to quantum gravity
- Predicting Λ
- Conclusions
- Problems?

The cosmological constant

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \;=\; 8\pi G T_{\mu\nu}$$

- *One* explanation for dark energy.
- Observations suggest $\Lambda = 10^{-120}$ (Planck units).

Why is Λ so very small, but not zero?

The causal set approach to quantum gravity

- Spacetime is a discrete structure whose elements are related only by a partial order.
- Causal information and a volume measure provide all the information about a spacetime:

$$\text{Order} + \text{Number} \rightarrow \text{Geometry}$$

Causal sets

A *causal set* (causet) is a locally finite partially ordered set, i.e. a set C endowed with a binary relation $\prec$ satisfying

1. transitivity: if $x \prec y$ and $y \prec z$ then $x \prec z$, $\forall x, y, z \in C$,
2. acyclicity: $x \not\prec x$,
3. local finiteness: $\forall x, z \in C$ the set $\{y \mid x \prec y \prec z\}$ of elements is finite.

Sprinkling

Select points from a Lorentzian manifold $\mathcal{M}$ (e.g. Minkowski spacetime) randomly using a Poisson process:

- divide $\mathcal{M}$ into small boxes of volume $\mathcal{V}$
- place a point independently in each volume with probability $\mathcal{V}/\mathcal{V}_{fund}$
- the Poisson process is the limit of this process as $\mathcal{V} \rightarrow 0$

The causal relations on $\mathcal{M}$ give rise to the partial order on the causal set.

Number and Volume

More precisely, the probability of sprinkling n points into a volume V is:

$$P(n) = \frac{(\rho V)^n e^{-\rho V}}{n!}$$

where ρ is a fundamental density.

- on average ρV points are sprinkled into a volume V
- fluctuations are of the order $\sqrt{\rho V}$

In the continuum limit of a causal set the volume approximately corresponds to the number of elements.

Dynamics

- Elements of a causal set are ‘born’ sequentially $\Rightarrow$ the number of elements N acts as a time parameter.
- Sum-over-histories approach: A quantum causal set dynamics will involve a sum over causal sets with a fixed number of elements N.

Everpresent Λ

- *Assume* unknown dynamics can be approximated by a gravitational path integral.
- Holding N fixed in sum-over-causets corresponds to holding V fixed in path integral

$\Rightarrow$ 'Unimodular gravity'

- In the unimodular modification of GR V and Λ are conjugate i.e there is an uncertainty relation $\Delta V \Delta\Lambda \sim 1$

$\Rightarrow$ if V is fixed then Λ is undetermined.

Everpresent Λ

Recall in causal set theory for fixed N there are fluctuations in the volume of order $\sqrt{N} \sim \sqrt{V}$ (taking $\rho = 1$).

$$\Delta\Lambda \sim \frac{1}{\Delta V} \sim V^{-1/2}$$

- *Assume* the value of the cosmological constant is driven toward zero i.e. $\langle\Lambda\rangle = 0$.
- Λ will not be exactly zero - will have fluctuations of order $V^{-1/2}$.

Λ today

Estimate the volume of the observable universe to be

$$V \sim (H^{-1})^4,$$

where H^{-1} is the Hubble radius.

$$\Rightarrow \Lambda \sim H^2 \sim 10^{-120}$$

in agreement (somewhat surprisingly?) with current observations.

Conclusions

- A heuristic argument gives a fluctuating cosmological constant $\Lambda \sim \pm 1/\sqrt{V}$.
- Agrees with observation if we live in 4 dimensions.
- A more complete argument requires an as yet unknown quantum causal set dynamics.
- Relates the size of the observable universe and the size of the cosmological constant - one less problem to solve.

Problems?

- What exactly are V and Λ?
  - We can interpret V at a point x as the past of x i.e. the number of ancestors of x in the causal set.
  - Λ may be an action per spacetime element.
- Λ in Einstein's equation *is* constant $\rightarrow$ a more concrete phenomenological model involves modification of Friedmann equations.
- Spatially inhomogeneous Λ leads to anisotropies in the CMB strongly constraining the number of elements per Planck volume in causal set theory.