

# Complementarity and Information in “Delayed-choice for Entanglement Swapping”\*

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*Building on Peres’ idea of “Delayed-choice for entanglement swapping” we show that even the degree to which quantum systems were entangled can be defined after they have been registered and may even not exist any more. This does not arise as a paradox if the quantum state is viewed as just a representative of information. Moreover such a view gives a natural quantification of the complementarity between the measure of information about the input state for teleportation and the amount of entanglement of the resulting swapped entangled state.*

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## 1. INTRODUCTION

Entangled systems display one of the most interesting features of quantum mechanics—their joint state is not separable regardless of their spatial separation. It is still sometimes believed that for obtaining entangled states quantum systems *necessarily* need to interact (dynamically) with one another, either directly, or indirectly via other particles.

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\*This Paper is Dedicated to Prof. Asher Peres on the Occasion of his 70th Birthday.

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Yet, an alternative possibility to obtain entanglement is to make use of projection of the state of two particles onto an entangled state. The procedure is known as “entanglement swapping” and can be seen as quantum teleportation of the state that itself is entangled.<sup>(1,2)</sup> It was experimentally demonstrated for the first time in 1998.<sup>(3)</sup> In the experiment two pairs of entangled photons 0–1 and 2–3 are produced and one photon from each of the pairs is sent to two separated observers, say photon 0 is sent to Alice and photon 3 to Bob, as is schematically shown in Fig. 1. The other photons, 1 from the first pair and 2 from the second pair, are sent to the third observer, Victor. He subjects photons 1 and 2 to a Bell-state measurement, by which photons 0 and 3 become automatically entangled. This requires the entangled photons 0 and 3 neither to come from a common source nor to have interacted in the past.

A seemingly paradoxical situation arises, if - as suggested by Peres<sup>(4)</sup> in his paper entitled “*Delayed-choice for entanglement swapping*”—“entanglement is produced *a posteriori*, after the entangled particles have been measured and may even no longer exist”. A similar situation was considered also by Cohen.<sup>(5)</sup> In such a scenario particles 0 and 3 are detected before the Bell-state measurement has been performed. This seems paradoxical, because Victor’s measurement projects photons 0 and 3 into an entangled state *after they have been registered*. Furthermore, Victor is even free to choose the kind of measurement he wants to perform on photons 1 and 2. Instead of a Bell-measurement he could also measure the polarization of these photons individually which would result in a well defined

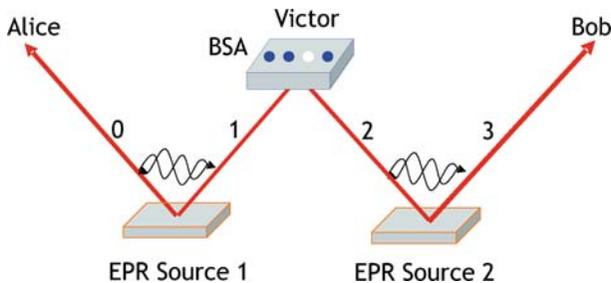


Fig. 1. Scheme of entanglement swapping. Two pairs of entangled particles 0–1 and 2–3 are produced by two Einstein–Podolsky–Rosen (EPR) sources. One particle from each of the pairs is sent to two separated observers, say particle 0 is sent to Alice and particle 3 to Bob. The other particles 1 and 2 from each pair are sent to Victor who subjects them to a Bell-state analyser (BSA), by which particles 0 and 3 become entangled although they have never interacted in the past.

polarization for photons 0 and 3, i.e., a separable product state. Therefore, depending on Victor's later measurement, Alice's and Bob's earlier results indicate that photons 0 and 3 were either entangled or not.

In this article we put this paradoxical situation to its extreme to show that even the *degree* to which the particles were entangled can be defined after the particles have been registered. This is because Victor could choose a measurement for photons 1 and 2 projecting them into an *arbitrarily partially* entangled state after photons 0 and 3 have been detected. Therefore, depending on the degree of entanglement of the states into which photons 1 and 2 are projected in Victor's later measurement, Alice's and Bob's earlier results correspond to a specific degree of entanglement of photons 0 and 3. Furthermore, this degree of entanglement is specified by the amount of information about the state of photon 1 or 2, which is gained in Victor's measurement of photons 1 and 2. Using an information-theoretic description we give a quantitative *complementarity relation* for the degree of entanglement of photons 0 and 3 and the amount of information about the state of photon 1.

## 2. RELATIONAL PROPOSITIONS AND ENTANGLEMENT

Recently, delayed-choice entanglement swapping in the spirit of the proposal by Peres<sup>(4)</sup> was experimentally demonstrated using polarization-entangled photons.<sup>(6)</sup> In such experiments, a Bell-state measurement is realized by overlapping two photons on a balanced (50:50) beamsplitter and analyzing their distribution in the output ports. Specifically, for a projection on the spin-singlet state  $|\psi^-\rangle$  the two photons exit in different ports of the beamsplitter. By including two 10 m optical fibre delays for both outputs of the Bell-state measurement photons 1 and 2 hit the detectors delayed by about 50 ns with respect to detection of photons 0 and 3 (Fig. 2). Importantly, it was shown that the observed fidelity of the entangled state of photons 0 and 3 matches the fidelity in the nondelayed case within experimental errors. This indicates that relative temporal order of Alice's and Bob's events on one hand, and Victor's events on the other hand is irrelevant. It also strengthens the view that what here matters is bringing lists of Alice's, Bob's and Victor's measurement results into an appropriate *relation*.<sup>(6-8)</sup> Alice and Bob can divide their own lists of locally completely random measurement results into many different subsets of results that correspond to different possible Victor's measurements. Which events fall into which subset they might learn in the future from Victor's measurement results. And it is independent of whether Victor's measurement has been performed before or even after Alice's and Bob's

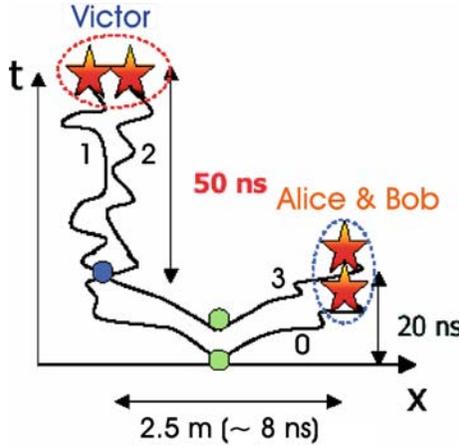


Fig. 2. Space-time diagram for delayed choice entanglement swapping experiment reported in Ref. 6. Alice’s and Bob’s detector were located next to each other. The time travel of photons 0 and 3 from the source to these detectors was equal to about 20 ns. By including additional optical fiber delays for both outputs of the Bell-state analyser photons 1 and 2 hit Victor’s detectors delayed by about 50 ns. Victor and the pair Alice and Bob were separated by about 2.5 m, corresponding to luminal traveling time of 8 ns between them.

measurements. Because the quantum state is no more than the most compact representative of such lists,<sup>(9)</sup> it is conceptually allowed to say that particles can become entangled even after they already have been registered.

In the present work we will consider entanglement swapping implemented in terms of states of spin-1/2 particles. Hence the particle pairs are prepared in one of the four Bell states, e.g., in the singlet state

$$|\psi^-\rangle_{01} = \frac{1}{\sqrt{2}}(|z+\rangle_0|z-\rangle_1 - |z-\rangle_0|z+\rangle_1), \tag{1}$$

where, e.g.,  $|z+\rangle_0$  describes the state “spin up” of particle 0 along z-direction. This entangled state contains no information on the spin of the individual particles; it only indicates that the two particles will give the opposite results if their spins are measured along the same directions. Thus, as soon as a measurement on particle 0 projects it, say, onto state  $|z+\rangle_0$  the state of the

other one is determined to be  $|z-\rangle_1$ , and vice versa. Thus the correlations for measurements of spin of the two particles along  $z$  can be represented by the proposition:

(i) “*The two particles have the opposite spin along the  $z$ -axis*”.

Because the state (1) is invariant under identical local unitary transformations the correlations for spin measurements along  $x$  can be represented by the similar proposition:

(ii) “*The two particles have the opposite spin along the  $x$ -axis*”.

The two propositions fully define the entangled state (1). This can be seen as a consequence that two bits of information are available to define a two-qubit state.<sup>(10)</sup> The truth value of the proposition “*The two particles have the opposite spin along the  $y$ -axis*” follows from the truth values of the propositions (i) and (ii). This is because a joint eigenstate of spin products  $\sigma_x^1\sigma_x^2$  and  $\sigma_y^1\sigma_y^2$  is also the eigenstate of  $\sigma_z^1\sigma_z^2 = -(\sigma_x^1\sigma_x^2)(\sigma_y^1\sigma_y^2)$ . Here, e.g.,  $\sigma_x^1$  is spin along  $x$  of particle 1. In general, the four Bell states can be seen as representation of the four possible two-bit combinations (true–true, true–false, false–true, false–false) of the truth values of the propositions (i) and (ii).<sup>(11)</sup>

Initially, in the entanglement swapping experiment, the system is composed of two independent entangled pairs in the overall state:

$$|\psi_{\text{total}}\rangle = |\psi^-\rangle_{01}|\psi^-\rangle_{23}. \quad (2)$$

If Victor now subjects particles 1 and 2 to a measurement in a Bell-state analyzer (BSA), and if he finds them in the state  $|\psi^-\rangle_{12}$ , then particles 0 and 3 measured by Alice and Bob, respectively, will be also in the maximally entangled state  $|\psi^-\rangle_{03}$ . The reason for this can be seen using the logic of propositions. We had initially prepared the pair of particles 0 and 1 in the state  $|\psi^-\rangle_{01}$  and the pair 2 and 3 in the state  $|\psi^-\rangle_{23}$  which means that for measurements along the same directions particles in both the pair 0–1 and in the pair 2–3 will give opposite results. Because we observe particles 1 and 2 in the state  $|\psi^-\rangle_{12}$  we know that whatever the result measured on particle 1 is, the one measured on particle 2 must be opposite, if they are measured along the same but otherwise arbitrary directions. This is only possible if results measured on particles 0 and 3 will be opposite with respect to any chosen measurement basis, which is the actual symmetry of the spin singlet state. This implies that the final state of particles 0 and 3 is  $|\psi^-\rangle_{03}$ .

We now suppose that Victor chooses a measurement for particles 1 and 2 projecting them into one of the following four (orthogonal) states:

$$|\psi_{\alpha}^{+}\rangle_{12} = \alpha|z+\rangle_1|z-\rangle_2 + \beta|z-\rangle_1|z+\rangle_2, \quad (3)$$

$$|\psi_{\alpha}^{-}\rangle_{12} = \beta|z+\rangle_1|z-\rangle_2 - \alpha|z-\rangle_1|z+\rangle_2, \quad (4)$$

$$|\phi_{\alpha}^{+}\rangle_{12} = \alpha|z+\rangle_1|z+\rangle_2 + \beta|z-\rangle_1|z-\rangle_2, \quad (5)$$

$$|\phi_{\alpha}^{-}\rangle_{12} = \beta|z+\rangle_1|z+\rangle_2 - \alpha|z-\rangle_1|z-\rangle_2 \quad (6)$$

with *arbitrary* value of  $\alpha$  such that  $\alpha, \beta \in \mathbb{R}$  and  $\alpha^2 + \beta^2 = 1$ . For the cases  $\alpha = \beta = (1/\sqrt{2})$  ( $\alpha = 1$  or  $0$ ), this projection reduces to the previous Bell-state measurement (to the measurement of spins along  $z$  of particles 1 and 2 individually).

Including Eqs. (1) and (2) and rearranging the resulting terms by expressing states of particles 1 and 2 in the basis of the states (3–6) leads to

$$|\psi_{\text{total}}\rangle = \frac{1}{2} \left[ |\psi_{\alpha}^{+}\rangle_{03} |\psi_{\beta}^{+}\rangle_{12} - |\psi_{\alpha}^{-}\rangle_{03} |\psi_{\beta}^{-}\rangle_{12} + |\phi_{\alpha}^{+}\rangle_{03} |\phi_{\beta}^{+}\rangle_{12} - |\phi_{\alpha}^{-}\rangle_{03} |\phi_{\beta}^{-}\rangle_{12} \right]. \quad (7)$$

Each of the four states (3–6) show either perfect correlations or perfect anticorrelations for measurements of spins along  $z$ . Thus each of them can be described by a definite truth value of proposition (i). Yet, they are only partially entangled and we cannot assert them a definite truth value also for proposition (ii). Instead, for example, the representation of the state  $|\psi_{\alpha}^{-}\rangle_{12}$  in the eigenbases of spins along  $x$  is given by

$$\begin{aligned} |\psi_{\alpha}^{-}\rangle_{12} = & \frac{1}{2}(\alpha - \beta) \left( |x+\rangle_1|x+\rangle_2 - |x-\rangle_1|x-\rangle_2 \right) \\ & + \frac{1}{2}(\alpha + \beta) \left( |x+\rangle_1|x-\rangle_2 - |x-\rangle_1|x+\rangle_2 \right). \end{aligned} \quad (8)$$

Thus, the correlations for spin measurements along  $x$  are not perfect.

### 3. A MEASURE OF INFORMATION

If quantum physics is to be viewed as a science about what we can say about possible measurement results and if, as we just have seen, we cannot assert definite truth values to all possible propositions, then we suggest to introduce the notion of information, or our knowledge about these truth values. It is then natural to refer to the amount of information on the truth values of propositions.

Elsewhere we introduced an appropriate measure of information to quantify this knowledge in quantum mechanics.<sup>(12)</sup> If we denote the probability for spins of the particles to be equal along  $x$  by  $p_{xx}^+$  and the probability for them to be opposite by  $p_{xx}^-$ , the measure is given by

$$I_{xx} = (p_{xx}^+ - p_{xx}^-)^2. \tag{9}$$

Similarly, information about the spins of two particles along  $z$  directions is given by  $I_{zz} = (p_{zz}^+ - p_{zz}^-)^2$ . In the case of the four partially entangled states (3–6) we have  $I_{zz} = 1$  and  $I_{xx} = 4\alpha^2\beta^2$  and for the maximally entangled states  $I_{xx} = I_{zz} = 1$ .

Suppose Victor subjects particles 1 and 2 to a measurement in a generalized BSA and he finds them in the not maximally entangled state  $|\psi_\alpha^-\rangle_{12}$ . Then, according to Eq. (7) particles 0 and 3 will be in the also not maximally entangled state  $|\psi_\beta^-\rangle_{03}$  (which is defined by inserting  $\beta$  instead of  $\alpha$  in the expression (4)). The reason for this can be seen even without involving Eq. (7) by logically combining propositions and using our measure of information. Because we observe particles 1 and 2 in the state  $|\psi_\alpha^-\rangle_{12}$  our knowledge on possible outcomes for measurements of spins of particles 1 and 2 along directions  $z$  and  $x$  is represented by  $I_{zz}^{12} = 1$  and  $I_{xx}^{12} = 4\alpha^2\beta^2$ , respectively. We note again that with this we mean our a priori information about what outcome will occur if the corresponding experiment is performed. Initially we had prepared the pair of particles 0 and 1 in the state  $|\psi^-\rangle_{01}$  and the pairs 2 and 3 in the state  $|\psi^-\rangle_{23}$  which means that for the pair 0–1 we have  $I_{zz}^{01} = I_{xx}^{01} = 1$  and similarly for the pair 2–3 we have  $I_{zz}^{23} = I_{xx}^{23} = 1$ . By combining propositions in a logical chain, which in the case of incomplete knowledge of joint truth values corresponds to a multiplication of the corresponding measures of information, we obtain

$$I_{zz}^{03} = I_{zz}^{01} \cdot I_{zz}^{12} \cdot I_{zz}^{23} = 1, \tag{10}$$

$$I_{xx}^{03} = I_{xx}^{01} \cdot I_{xx}^{12} \cdot I_{xx}^{23} = 4\alpha^2\beta^2 \tag{11}$$

for the measures of information for the pair of particles 0 and 3 for measurements along  $z$  and  $x$ , respectively. These values indeed correspond to the state  $|\psi_\beta^-\rangle_{03}$ . Interestingly, we obtained this by using only operations on measures of information without involving the standard quantum formalism.

In Ref. 13 we suggested to describe the total information contained in correlations of two particles as the sum over the individual measures of information  $I_{zz}$  and  $I_{xx}$  carried in a set of spin measurements of the two particles along  $z$  and  $x$  directions, i.e.,

$$I_{\text{corr}} \equiv I_{zz} + I_{xx}. \tag{12}$$

Here the value of  $I_{\text{corr}}$  is maximized over all possible choices of local coordinate systems (i.e., local  $z$  and  $x$  directions) for particles 1 and 2. The measurements of spin products along  $z$  and along  $x$  are *mutually complementary for product states*. That is, for any product state (and, more general, for any separable state) complete knowledge contained in one of the observations in Eq. (12) excludes any knowledge in the other observation. For example, for the case of the product state  $|z+\rangle_1|z+\rangle_2$  one has full information about spins along  $z$  ( $I_{zz} = 1$ ) at the expense of complete lack of information about spins along  $x$  ( $I_{xx} = 0$ ). This suggests that  $I_{\text{corr}}$  can be considered as an information-theoretic measure of entanglement in the spirit of Schrödinger's "entanglement of our knowledge"<sup>(9)</sup>: The value of  $I_{\text{corr}}$  is smaller than or equal to 1 for separable states, whereas it is 2 for the maximally entangled state. In the intermediate case of a partially entangled state  $|\psi_\alpha^-\rangle_{03}$  this value is between 1 and 2 bits of information, i.e.,

$$I_{\text{corr}}^{03} = 1 + 4\alpha^2\beta^2. \quad (13)$$

Quantum entanglement displays one of the most distinct features of quantum mechanics against the classical world—i.e. the conflict with local realism—as quantified by violation of Bell's inequalities.<sup>(14)</sup> An example of such an inequality is the Clauser–Horne–Shimony–Holt<sup>(15)</sup> (CHSH) inequality

$$S = |E(\phi, \tilde{\phi}) + E(\phi, \tilde{\phi}') + E(\phi', \tilde{\phi}) - E(\phi', \tilde{\phi}')| \leq 2, \quad (14)$$

where 2 on the right-hand side is the local realistic limit,  $S$  is the "Bell parameter", and  $E(\phi, \tilde{\phi})$  is the correlation function for measurements of two particles with local measurement setting  $\phi$  for the first particle and  $\tilde{\phi}$  for the second particle, respectively.

In Ref. 13 we showed that the condition that *more than one bit* of information is contained in correlations of measurements on a pair of particles is a *necessary* and *sufficient* condition<sup>(16)</sup> for violation of the CHSH inequality by the pair of particles. Mathematically, we have

$$I_{\text{corr}} > 1 \Leftrightarrow S > 2. \quad (15)$$

It can be shown that the relation between the Bell parameter and the amount of information contained in correlations is given by  $I_{\text{corr}} = S^2/4$ .<sup>(13)</sup>

#### 4. A COMPLEMENTARITY RELATION FOR INFORMATION

In Ref. 3 entanglement swapping was first experimentally demonstrated, but the low photon-pair visibility prevented a violation of a Bell's inequality for photons 0 and 3, which is an important test to confirm the quantum nature of teleportation. This is because the violation of Bell's inequalities for the measurements on photons 0 and 3 indicates that no significant information about the state of teleported photon 1 was gained in the teleportation procedure and, consequently, the teleportation is of high fidelity. This again can be seen from our information-theoretical considerations. Suppose that Victor finds particles 1 and 2 in the state  $|\psi_{\alpha}^{-}\rangle_{12}$ . Then we know that particle 1, if it is measured along  $z$ , will give result  $+1$  with probability  $\beta^2$  and result  $-1$  with probability  $\alpha^2$ . This knowledge about the individual properties of particle 1 can be defined as given by (this is analog of definition (9); see also Ref. 12)

$$I_{\text{ind}}^1 = (p_z^+ - p_z^-)^2 = (\beta^2 - \alpha^2)^2, \quad (16)$$

where, e.g.,  $p_z^+$  is the probability to observe result  $+1$  for measurement of spin of particle 1 along  $z$ .

We now give a “complementarity relation” for the amount of information contained in individual properties of particle 1 and the amount information contained in correlations, or joint properties, of the pair of particles 0 and 3. By summing Eqs. (13) and (16) we obtain

$$I_{\text{ind}}^1 + I_{\text{corr}}^{03} = (\beta^2 - \alpha^2)^2 + 1 + 4\alpha^2\beta^2 = 2. \quad (17)$$

Note that for symmetry reasons this is also true with particle 2 instead of 1. Equation (17) shows that if the pair of particles 0 and 3 violate the CHSH inequality, that is, if the particles carry more than one bit of information in their joint properties ( $I_{\text{corr}}^{03} > 1$ ), then information about individual properties of particle 1 will be incomplete ( $I_{\text{ind}}^1 < 1$ ), and vice versa. In the extreme case of maximal violation of the CHSH inequality ( $I_{\text{corr}}^{03} = 2$ ) no information is available to define properties of particle 1. Consequently, the teleported state is of perfect fidelity.

One can conjecture that the complementary relation in the form of the inequality  $I_{\text{ind}}^1 + I_{\text{corr}}^{03} \leq 2$  is also valid for more general type of measurements (see also Ref. 17). Then the inequality sign can be understood as coming from the possibility that information can flow out of the systems into the environment.

If Alice and Bob can only communicate via classical channel, they cannot transfer an arbitrary quantum state with a fidelity of one (here we

assume the standard teleportation scheme Alice has also access to Victor's measurement station). Instead, on the basis of her measurement result Alice can make a state estimation of the input state and send this information to Bob who can reconstruct the state on the basis of her result. The fidelity  $f$  of this "classical teleportation" is then defined as the overlap between the input state  $|\psi_{\text{in}}\rangle$  and the state estimated  $\rho_{\text{est}}$ , i.e.,  $f = \langle \psi_{\text{in}} | \rho_{\text{est}} | \psi_{\text{in}} \rangle$ .

Somewhat lengthy but otherwise straightforward calculation shows that fidelity of state estimation of particle 1, that is based on the measurement defined by eigenstates (3–6) is given by  $f = (2/3)(1 - \alpha^2\beta^2) = (1/2) + (I_{\text{ind}}^1/6)$ . Here  $f$  is averaged over all possible input states of particle 1. Note that while  $f$  refers to our knowledge about the pre-measurement state of particle 1,  $I_{\text{ind}}^1$  corresponds to our knowledge about the post-measurement properties of this particle. Thus, if the state of particle 1 after the measurement is completely undefined ( $I_{\text{ind}}^1 = 0$ ), the fidelity is  $f = 1/2$  just as for the random choice. If, in contrast, the measurement projects particle 1 into a well-defined (pure) state ( $I_{\text{ind}}^1 = 1$ ), the fidelity achieves the value of  $f_{\text{cl}} = 2/3$ . This is known as the classical limit for teleportation and is equal to the minimum fidelity of teleportation such that it can still be considered as genuine quantum.

The high-fidelity teleportation and violation of the CHSH inequality for measurements on photons 0 and 3 was recently observed in the entanglement swapping experiment.<sup>(6)</sup> The value for  $S$  obtained was  $S = 2.421 \pm 0.091$  which clearly violates the local realistic limit of 2 by 4.6 standard deviations as measured by the statistical error. Such a strong violation of the CHSH inequality excludes a principal possibility of the use of Alice's measurement result to classically reconstruct the teleported state at Bob's side with high fidelity (e.g., via some "hidden mechanism"). Since the experimental value for  $S$  corresponds to  $I_{\text{corr}}^3 = S^2/4 = 1.465$ , the maximum amount of information  $I_{\text{ind}}^1$  about particle 1 that could have been gained that way is bounded by the complementarity relation (17):  $I_{\text{ind}}^1 \leq 0.536$ . This results in a fidelity limit of  $f = 0.589$ , which is clearly lower than the classical limit of  $f_{\text{cl}} = 2/3 = 0.666$ .

## 5. CONCLUSIONS

In Peres words: "if we attempt to attribute an objective meaning to the quantum state of a single system, curious paradoxes appear: quantum effects mimic not only instantaneous action-at-a-distance but also, as seen here, influence of future actions on past events, even after these events have been irrevocably recorded". In contrast, there is never a paradox if

the quantum states is viewed as to be no more than a representative of our information. Furthermore such a view provides us with both conceptually and formally much simpler approach. This we demonstrate here for the entanglement swapping experiment by deriving a quantitative complementarity relation between the measure of information about the input state for teleportation and the amount of entanglement of the resulting swapped entangled state.

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